TechnoLab

Truss HomeLab Series

Experiment T3: Forces in a 7-bar Truss – Type I

Note: If the Pixi Frame has not been set up ahead of this experiment, refer to the Document "Setting up the Pixi Frame for Mechanics Experiments" and go to the Section noted for Truss HomeLab Experiment T3.

Aims: This experiment is designed to allow students to observe, measure and compare the static force equilibrium conditions at the connection nodes of a 7-bar truss (members of equal initial length and stiffness, see Figure 1) to loading applied at one or more "free" nodes.

The actual loads at the nodes and their orientation are to be selected by the instructor and noted in Table I below to be consistent with the Tables for these Load Cases as depicted on Pages 6, 7 and 8).

Table I: Load Values and Orientation of Load for Selected Nodes of Truss to be investigated.

	Node #	Approx. Load	Node #	Load	Node #	Load
Case 1	4					
Case 2	3					
Case 3	4		3			

The forces in the bars are to be evaluated from the nominal stiffness of each (TBA). The theoretical forces in each bar, (using the method of joints and a computer program), are then to be compared with the results obtained from measurements.

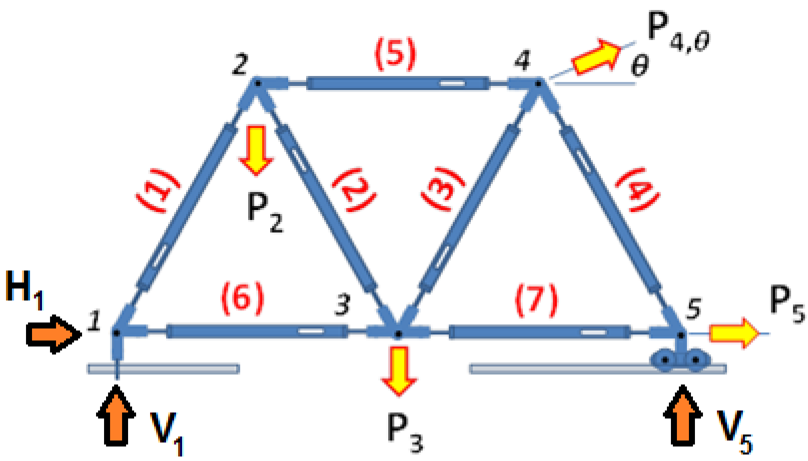


Figure 1: Load Cases of 7-bar truss to be investigated.

Learning Outcomes:

After performing this experiment students will be able to:

(i) Use the Spreadsheet supplied (based on the method of joints), and the computer program noted, to compare the predicted forces in each member of a 7-bar truss induced by the applied loads with the experimentally obtained values of these forces.

(ii) Reinforce their understanding of the equilibrium of co-planar forces in the context of the equilibrium of applied loads and reactions acting on a 7-bar truss.

(iii) Reinforce their understanding of the equilibrium of co-planar forces in the context of the equilibrium of forces at the pin-joints of a 7-bar truss.

(iv) Appreciate the role played by a member's characteristic stiffness in the stiffness method in the evaluation of internal actions from the individual member elongations/contractions.

Equipment/Resources Required:

(i) Pixi with window frame in "landscape" configuration

(ii) 7-bar Truss (equal member stiffness and length)

(iii) Set of chrome/stainless steel bearing balls (weight forces), load buckets and pulleys

(iv) A digital camera (phone cameras are suitable)

(v) xyRectify software on a suitable Notebook, Laptop or PC (optional)

or TechnoLab™ "T-Force" image processing software (TBA)

(vi) Instruction sheets and support material supplied by TechnoLab for this experiment group

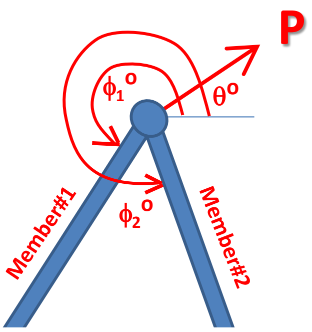
Introduction:

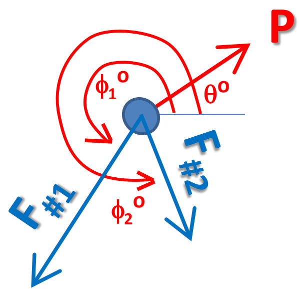
In this experiment we will investigate the internal forces induced in a 7-bar truss from application of a pattern of point loads applied to free joints and/or the roller support, from evaluation of the elongation/contraction of each model truss member and its nominal stiffness. Comparison of the experimental results will then be made with evaluation of the member forces using the "method of joints" via a spreadsheet and independently using a computer Truss analysis package.

Basic Theory:

Consider any joint of a 7-bar truss of identical prismatic members (cross-section and length the same for all 7 members, as in Figure 1, made of linear elastic material (E, ν).

Suppose that at a particular joint in the truss, there are two unknown member forces, (identified as $F_{\#1}$ and $F_{\#2}$ for "local" members#1 and #2) and that the resultant of any applied load, reaction and known member force(s) acting on that node sum to a point load of P, as depicted in Fig. 2.





$$\sum F_x = 0; \quad Pcos(\theta) + F_1\cos(\phi_1) + F_2\cos(\phi_2)$$

$$\sum F_y = 0; \quad Psin(\theta) + F_1\sin(\phi_1) + F_2\sin(\phi_2)$$

Figure 2: Application of "Method of Joints" at a Node of a 7-bar Truss with two unknown member forces

From Fig. 2, it can be seen that because member forces F_1 and F_2 of the 7-bar truss are in equilibrium with resultant load P at the joint of interest, then the horizontal and vertical components of all forces acting on this joint must separately sum to "zero".

Assuming that the forces in each truss member, F_1 and F_2, are tensile, this leads to two simultaneous linear equations in F_1 and F_2, as noted in Fig. 2.

These equations can be represented in matrix form as follows:

$$\begin{bmatrix} \cos(\phi_1) & \cos(\phi_2) \\ sin(\phi_1) & \sin(\phi_2) \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = -P \begin{Bmatrix} \cos(\theta) \\ \sin(\theta) \end{Bmatrix} \quad (1)$$

which can be solved to obtain:

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{P}{\sin(\phi_2)\cos(\phi_1) - sin(\phi_1)\cos(\phi_2)} \begin{bmatrix} -\sin(\phi_2) & \cos(\phi_2) \\ sin(\phi_1) & -\cos(\phi_1) \end{bmatrix} \begin{Bmatrix} \cos(\theta) \\ \sin(\theta) \end{Bmatrix} \quad (2)$$

$$\rightarrow \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{P}{\sin(\phi_2 - \phi_1)} \begin{Bmatrix} -\sin(\phi_2)\cos(\theta) + \cos(\phi_2)\sin(\theta) \\ \sin(\phi_1)\cos(\theta) - \cos(\phi_1)\sin(\theta) \end{Bmatrix} \quad (3)$$

$$\rightarrow \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{P}{\sin(\phi_2 - \phi_1)} \begin{Bmatrix} -\sin(\phi_2 - \theta) \\ \sin(\phi_1 - \theta) \end{Bmatrix} = P \begin{Bmatrix} -\frac{\sin(\phi_2 - \theta)}{\sin(\phi_2 - \phi_1)} \\ \frac{\sin(\phi_1 - \theta)}{\sin(\phi_2 - \phi_1)} \end{Bmatrix} \quad (4)$$

Example 1: P (= P3) = 2 N acting vertically downwards at Node#3 of the 7-bar Truss.

From Symmetry, the two vertical reactions acting at Node#1 and Node#5 are both 1 N upwards as shown in Fig. 3. Node#1 and Node#5 are identified as nodes at which there are two unknown member forces (original Members (1) and (6) denoted as local Members #1 and #2 in Fig. 3), with resultant load acting on the node from known member forces, externally applied loads and reactions (if applicable), here being the vertical upward reaction of 1 N.

Hence:

Ex.	ϕ_2	ϕ_1	θ	$F_1 = -P\frac{\sin(\phi_2-\theta)}{\sin(\phi_2-\phi_1)}$	$F_2 = P\frac{\sin(\phi_1-\theta)}{\sin(\phi_2-\phi_1)}$
1	60°	0°	90°	$= -1.\sin(-30°)\,/\,\sin(60°) = -(-1/2/\sqrt{3}/2) = \mathbf{1/\sqrt{3}}$	$= 1.\sin(-90°)\,/\,\sin(60°) = (-1/\sqrt{3}/2) = \mathbf{-2/\sqrt{3}}$

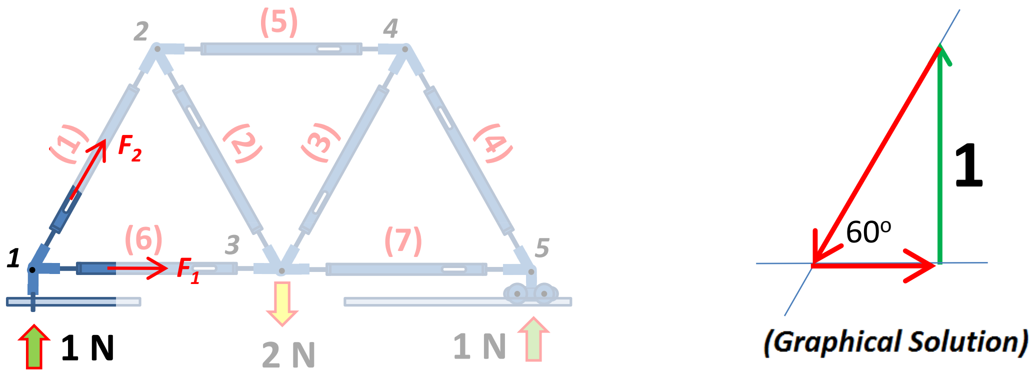


Figure 3: 7-bar truss with 2N downward vertical load acting at Node#3

Now with the force in Member (1) and (6) known we can see that we can progress to Node#2 as there would be two unknown member forces (Members (2) and (5)) where the force in Member (1) can be treated as an applied load P acting at Node#2 of $2/\sqrt{3}$ N at an angle θ = 60° as in Fig. 4.

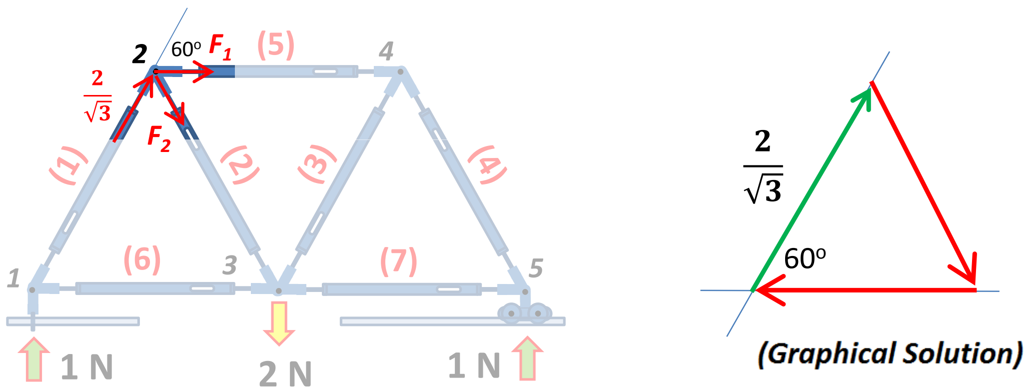
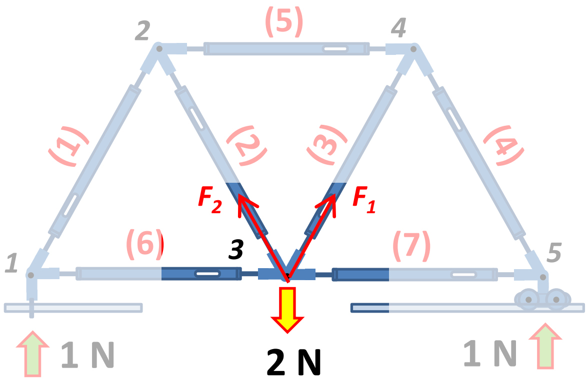


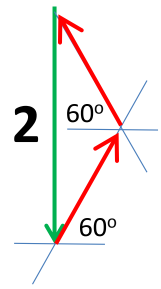
Figure 4: 7-bar Truss – Equilibrium at Node#2

Hence

Ex.	ϕ_2	ϕ_1	θ	$F_1 = -P\frac{\sin(\phi_2-\theta)}{\sin(\phi_2-\phi_1)}$	$F_2 = P\frac{\sin(\phi_1-\theta)}{\sin(\phi_2-\phi_1)}$
2	300°	0°	60°	$=-\frac{2}{\sqrt{3}}\sin(240°)\,/\,\sin(300°) =-\frac{2}{\sqrt{3}}(-\sqrt{3}/2/-\sqrt{3}/2)$ $= -\frac{2}{\sqrt{3}}$	$=\frac{2}{\sqrt{3}}\sin(-60°)\,/\,\sin(300°) =\frac{2}{\sqrt{3}}(-\sqrt{3}/2/-\sqrt{3}/2)$ $= \frac{2}{\sqrt{3}}$

We could have instead progressed to Node#3 using the Method of Joints, since because of symmetry Member (7) has the same force as Member (6), so that there are only two unknown member forces at Node#3, the member forces for Members (2) and (3). In addition, again because of symmetry, the forces in Members (2) and (3) should be equal.





(Graphical Solution)

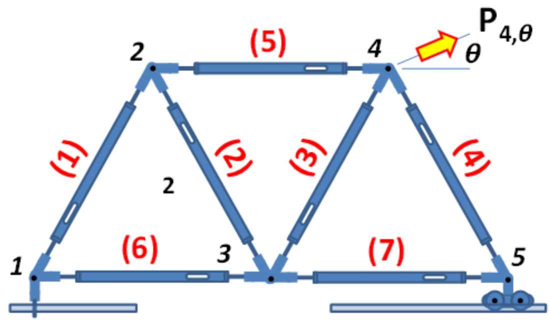
Check:

Ex.	ϕ_2	ϕ_1	θ	$F_1 = -P\frac{\sin(\phi_2-\theta)}{\sin(\phi_2-\phi_1)}$	$F_2 = P\frac{\sin(\phi_1-\theta)}{\sin(\phi_2-\phi_1)}$
3	120°	60°	-90°	$=-2\sin(210°)\,/\,\sin(60°) =-2\,(-\frac{1}{2}/\sqrt{3}/2)$ $= \frac{2\sqrt{3}}{3}$	$= 2\sin(150°)\,/\,\sin(60°) =2\,(\frac{1}{2}/\sqrt{3}/2)$ $= \frac{2\sqrt{3}}{3}$

Remaining forces can be inferred from Symmetry.

Experimental Investigations

Case 1: 7-bar Truss – Load @ Node#4



$P_{4,\theta}$	θ
As noted and Nominated below	
~200gm	0°
~400gm	0°
~200gm	30°
~400gm	30°

Place the window frame (with graticule transparency fixed on the outside of the polycarbonate sheet), onto the back side of the Pixi Frame and slide it close against this frame then fix it in position using the clasping arrangement.

Note the stiffness for the truss members being used for this experiment setup. (To be nominated at time of the class).

"Mark" the initial positions of the joints of the truss (define the initial deflected shape of this 7-bar truss under its self-weight), onto the transparency. Insert the removable opaque back-plane between the Window frame and the Pixi Frame. Take one or more photographs of the experimental setup, using an approved digital camera, taken far enough from the Pixi Frame to capture the full frame and as close as practical to the central normal of the plane of the Pixi Frame, with the experimental setup in focus. (The digital photographs can be used in combination with the "Simplified Photogrammetric Method" of Appendix 1 to provide a "direct" method for establishing individual member elongation/ contraction and hence their internal forces. Alternatively, **xyRectify** software for photogrammetric rectification of the images from which elongation/contraction of each member of the 7-bar truss can be measured, is also available upon request).

Measure the reactions at Node#1 and Node#3 using the Digital Scales provided.

Now attach a weight force of the approx. nominated value to Node 4. Use the digital scales provided to more accurately record the weight of the load mass being used. Arrange the pulley to allow loading at the nominated angle of inclination for the applied load at the "Node 4" of the 7-bar truss. Attach the load mass ring onto the load string ring using the "S" hook whilst holding the load mass from beneath. Gently transfer the weight of the load mass onto the loading string and observe the deflection produced at the joints of the 7-bar truss. Pull and release the load string gently to remove any residual friction in the pulley and load attachment system to the 7-bar truss.

Now remove the removable opaque back-plane from between the Window frame and the Pixi Frame. "Trace" the deflected position of the five connection nodes of the 7-bar truss, under the applied load onto the transparency and the orientation of the load string (slight departure from original). Estimate the node deflections, (difference in nodal positions with load applied and under self-weight), and evaluate elongation/contraction of each truss member.

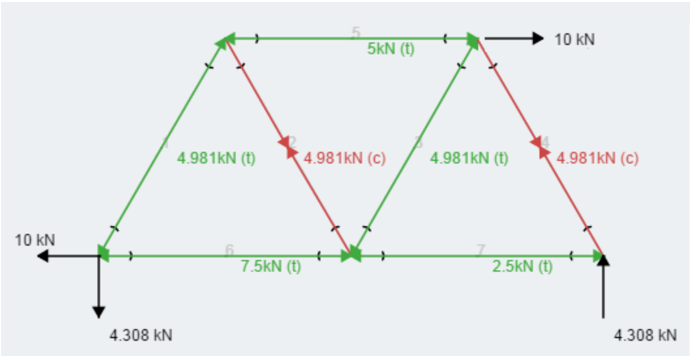
Re-insert the removable opaque back-plane between the Window frame and the Pixi Frame. Take another set of one or more photographs, taking care to observe the conditions for capturing these photographs described earlier.

The member forces for the 7-bar truss can then be determined from the nominal value of the member stiffness and the elongation/contraction due to the applied loading for each member as obtained using **xyRectify** software or the "Simplified Photogrammetric Method" of Appendix 1.

These values can then be compared with their theoretical counterparts using the spread-sheet **7-Bar_Truss.xls** supplied.

Alternatively, a check can be performed using on-line software packages, eg:

https://skyciv.com/free-truss-calculator/ (Free version: Force Evaluation only – no displacements)

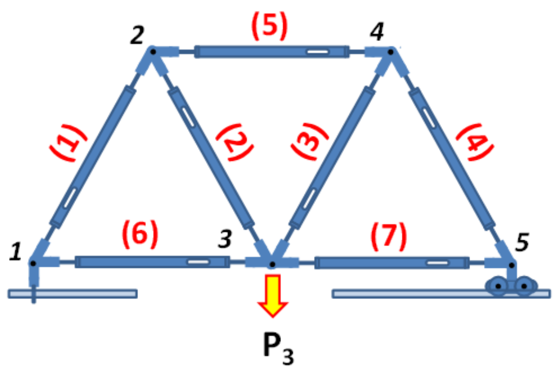


https://courses.cit.cornell.edu/arch264/calculators/example1.6/index.html (Neat Program)

http://www.federicobonfigli.com/EN/TrussSolver.aspx (Not always stable)

Case 2: 7-bar Truss – Load @ Node#3

P_3	θ
As noted and Nominated below	
~300gm	270°
~400gm	270°
~500gm	270°
~600gm	270°

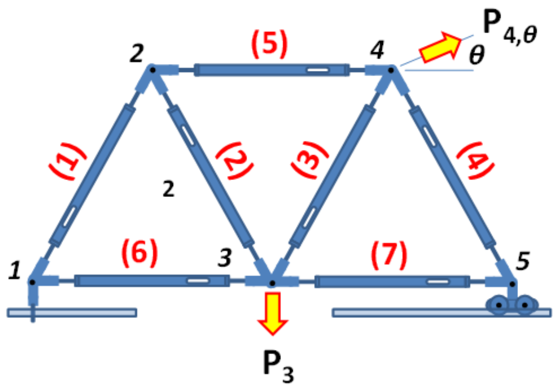


Repeat the procedure detailed for Case 1 for nominated vertical load *P_3* applied at Node#3.

Comment on the member force values obtained and accuracy of satisfaction of results to the condition of symmetry, (eg Accuracy of member force equality: $F_1 = F_4$; $F_2 = F_3$; $F_6 = F_7$; Accuracy of Reaction equality: $R_1 = R_5$).

Case 3: 7-bar Truss – Case 1 and 2 Loads Applied Simultaneously

$P_{4,\theta}$	θ
As noted for Case 1	
P_3	θ
As noted for Case 2	
	270°



Repeat the procedure detailed for Case 1 for load applied @ Node#4 simultaneously with nominated vertical load P_3 applied at Node#3.

Comment on the member force values obtained and satisfaction of equilibrium at the nodes.

Comment also on the sum of deflections for Load Case 1 and Load Case 2 at each node in comparison with the nodal deflections for Load Case 3 for each corresponding node of this truss.

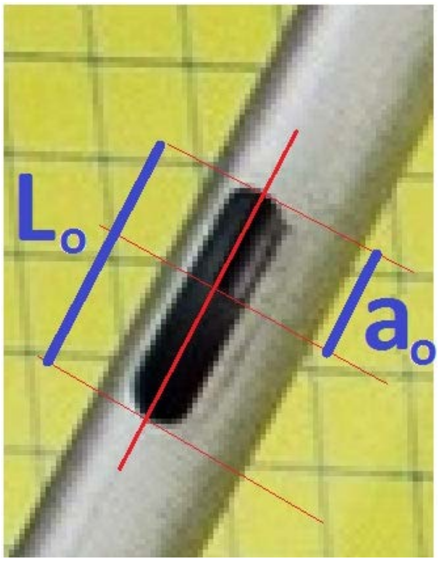
How do the sums of member forces for Load Case 1 and Load Case 2 in each member compare with the member forces for Load Case 3 for each corresponding member of this truss?

Appendix I: Simplified Photogrammetric Method for estimating a truss member's extension/contraction

Consider Member#1 before application of loading. Take a close-up photo of the slot of Member#1, clearly depicting the internal pointer and its position. Measure distances $\mathbf{L_o}$ and $\mathbf{a_o}$ on this photo (eg using a ruler or the digital callipers provided of the image on your phone if using a phone camera).

Member #1

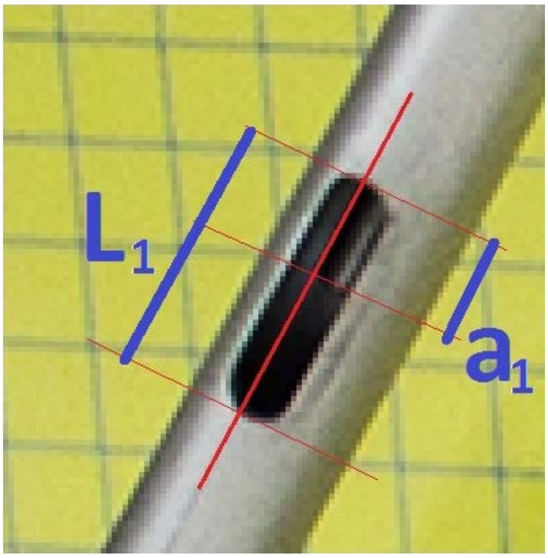
Before application of loading on Truss



Consider Member#1 **after** application of loading. Take a close-up photo of the slot of Member#1 that clearly depicts the internal pointer and its position. Measure distances $\mathbf{L_1}$ and $\mathbf{a_1}$ on this photo (eg using a ruler or the digital callipers provided of the image on your phone if using a phone camera).

Member #1

After application of loading on Truss



Since the actual slot length is 13.0 mm in each of the truss members, the position of the internal pointer from the edge of the slot is in the proportion **a/L** of the slot length, (irrespective whether the photo is rectified or not). The elongation of the member, δ_1, after application of the load becomes:

$$\delta_1 = 13.0\left(\frac{a_o}{L_o} - \frac{a_1}{L_1}\right)\,\text{mm}$$

(if –ve, then contraction)

Notes: